Finite key effects — in satellite quantum key distribution —





Outline

- 1. Introduction & overview
- 2. Modelling satellite QKD
 - operation description
 - finite block sizes
 - SatQuMA: optimised finite key length software
- 3. Applications
 - system performance
 - expected annual SKL
 - protocol performance
- 4. Summary of work



J. Sidhu, T. Brougham, D. McArthur, R. Pousa, D. Oi, arXiv:2012.07829.

Introduction & Overview

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¹ Front. Phys. 13(5), 130314 (2018).

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- some regions inaccessible free space links required

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² Satellite-to-ground QKD, Nature **549** 43 (2017). ³ Network over 4,600 km, Nature **589** 214 (2021).

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Distributed quantum technologies.



Modelling satellite QKD



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System link efficiency $\eta_{\rm link}^{\rm sys}$ characterises performance of SatQKD: satellite-OGS link efficiency at zenith.

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A baseline of $\eta_{\text{link}}^{\text{sys}} = 27 \text{ dB}$ is considered - empirical data from Micius.

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SatQKD operation (II)

General satellite overpass geometry for circular orbit of altitude *h*:



Single block: $SKL_{finite} = SKL(\{n_k^{\mu}, m_k^{\mu}\}),$

where $\{n_k^{\mu}, m_k^{\mu}\}$ = agglomerated counts without partitioning into sub-segments.

Three intensities μ_j with probabilities p_j , such that $\mu_1 > \mu_2 > \mu_3 = 0$:

Finite block secret key length (SKL)
$$\ell = \mathbf{s}_{X,0} + \mathbf{s}_{X,1}(1 - h(\phi_X)) - \lambda_{EC} - 6\log_2 \frac{21}{\epsilon_s} - \log_2 \frac{2}{\epsilon_c}$$

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Finite SKL determined from finite sample data block sizes

$$n_{\mathsf{X}(\mathsf{Z}),k}^{\pm} = \frac{e^k}{\rho_k} \left[n_{\mathsf{X}(\mathsf{Z}),k} \pm \delta_{n_{\mathsf{X}(\mathsf{Z}),k}}^{\pm} \right],$$

Correction terms: $\delta_Y^+ = \beta + \sqrt{2\beta y + \beta^2}, \quad \delta_Y^- = \frac{\beta}{2} + \sqrt{2\beta y + \frac{\beta^2}{4}}$

derived from inverse multiplicative Chernoff bounds with $\beta = \ln(1/\varepsilon)$.

⁵ Tight security bounds for decoy-state quantum key distribution, Sci. Rep. 10, 14312 (2020).

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Photon yield Vacuum yield Error cor

$$s_{\mathsf{X},1} = \frac{\tau_1 \mu_1 \left[n_{\mathsf{X},2}^- - n_{\mathsf{X},3}^+ - \frac{\mu_2^2 - \mu_3^2}{\mu_1^2} \left(n_{\mathsf{X},1}^+ - \frac{s_{\mathsf{X},0}}{\tau_0} \right) \right]}{\mu_1 (\mu_2 - \mu_3) - \mu_2^2 + \mu_3^2}$$

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Error correction

$$s_{X,0} \ge \tau_0 \frac{\mu_2 n_{X,\mu_3}^- - \mu_3 n_{X,\mu_2}^+}{\mu_2 - \mu_3}$$

Lower bound is tight when $\mu_3 \rightarrow 0$.

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Photon yield Vacuum yield Error correction

$$\lambda_{\text{EC}} = n_{\text{X}} h(Q) + n_{\text{X}} (1-Q) \log \left[\frac{(1-Q)}{Q} \right] \\
- \left(F^{-1}(\epsilon_c; n_{\text{X}}, 1-Q,) - 1 \right) \log \left[\frac{(1-Q)}{Q} \right] - \frac{1}{2} \log(n_{\text{X}}) - \log(1/\epsilon_c)$$

⁶ Fundamental finite key limits for one-way information reconciliation in quantum key distribution, Quant. Inf. Proc., 16280 (2017).

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 - trade-off block size with data quality.

























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Overpass transmission time optimisation is important

Low $\eta_{\text{link}}^{\text{sys}}$: construct keys using greatest amount of data (max Δt).

High $\eta_{\text{link}}^{\text{sys}}$: use only data around zenith

- better average QBER
- counters smaller raw key length and larger statistical uncertainties.

Maximise SKL over parameter space:

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Optimised finite key length, ℓ

$$\begin{array}{ll} \underset{p_{X}, \mu_{1}, \mu_{2}, p_{1}, p_{2}, \Delta t}{\text{maximize}} & s_{X,0} + s_{X,1}(1 - h(\phi_{X})) - \lambda_{\text{EC}} - 6\log_{2}\frac{21}{\epsilon_{s}} - \log_{2}\frac{2}{\epsilon_{c}} \\ & \text{subject to} & 0 < \{p_{X}, p_{j}\} < 1, \\ & 0 < \{\mu_{1}, \mu_{2}\} < 1, \\ & \mu_{1} > \mu_{2} > \mu_{3}, \\ & 0 < \Delta t \le t(10^{\circ}) \end{array}$$

Maximise SKL over parameter space:

Finite key effects in satellite quantum key distribution

Jasminder S. Sidhu,* Thomas Brougham,[†] Duncan McArthur,[‡] Roberto G. Pousa,[§] and Daniel K. L. Oi[¶] SUPA Department of Physics, University of Strathclyde, Glasgow, G4 0NG, United Kingdom (Dated: 26th April 2021)

Global quantum communications will enable long-distance secure data transfer, networked distributed quantum information processing, and other entanglement-enabled technologies. Satellite quantum communication overcomes optical fibre range limitations, with the first realisations of satellite quantum key distribution (SatQKD) being rapidly developed. However, limited transmission times between satellite and ground station severely constrains the amount of secret key due to finite-block size effects. Here, we analyse these effects and the implications for system design and operation, utilising published results from the Micius satellite to construct an empirically-derived channel and system model for a trusted-node downlink employing efficient BB84 weak coherent pulse decoy states with optimised parameters. We quantify practical SatQKD performance limits and examine the effects of link efficiency, background light, source quality, and overpass geometries to estimate long-term key generation capacity. Our results may guide design and analysis of future missions, and establish performance benchmarks for both sources and detectors.

Optimised finite key length

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Satellite Quantum Modelling & Analysis Software

https://github.com/cnqo-qcomms/SatQuMA.

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Maximise SKL over parameter space:



Satellite Quantum Modelling & Analysis Software

• toolkit to model satellite QKD

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- toolkit to model satellite QKD
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Variation in SKL with p_{ec} and QBER₁:

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Extraneous count probability



- p_{ec} changes vacuum yield s_{X,0}
- worse phase error/error correction
- *p*_{ec} at high η^{sys}_{link} gives zero SKL due to excessive QBER.

Variation in SKL with p_{ec} and QBER₁:

Extraneous count probability

Intrinsic QBER



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- worse phase error/error correction
- *p*_{ec} at high η^{sys}_{link} gives zero SKL due to excessive QBER.

- affects observed count rates
- SKL more robust to changes in QBER₁
- Focus on decreasing *p*_{ec}.

Variation in SKL with p_{ec} and QBER₁:



Improve background light suppression and detector dark counts over source fidelities and satellite alignment.

Expected annual finite key



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Annual key:
$$\overline{SKL}_{year} = N_{orbits}^{year} \frac{SKL_{int}}{L_{lat}}$$

where $N_{\text{orbits}}^{\text{year}}$ is the number of orbits per year, and L_{lat} is the longitudinal circumference along the line of latitude at the OGS location.

Expected annual finite key



$\eta_{\rm link}^{\rm sys}$	SKL _{int}	SKL _{year}
27 dB	$3.74 imes 10^{12}$ b	om 0.9131 Gb
30 dB	$1.52 imes 10^{12}$ b	om 0.3720 Gb
33 dB	$5.40 imes 10^{11}$ b	om 0.1318 Gb
37 dB	$8.75 imes 10^{10}$ b	om 0.0214 Gb
<u>40 dB</u>	$1.13 imes 10^{10}$ b	<u>om 0.0028 Gb</u>

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Multiple satellite passes

Data from several overpasses can be combined to improve SKL generation



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Systems with zero single-overpass SKL can generate key from M overpasses:

- $\ell_M \ge M \ell_1$ with diminishing improvement $\ell_{M+1} \ell_M$ with increasing M
- smaller estimation uncertainties from increased sample size
- greater latency leads to potential security vulnerabilities.

Protocol performance

Efficient BB84 performs better than standard BB84 in asymptotic regime.



Asymmetric BB84 delivers more finite key than symmetric BB84

- Improvement of 3 dB gives 7.6 times more annual key volume
- better sifting ratio and longer raw key length
- better handling of parameter estimation.



In summary ...

- 1. Numerical toolkit to benchmark system performance for SatQKD
- 2. SatQKD systems should prioritise background light suppression over higher intrinsic quantum signal visibilities or extending transmission
- 3. Efficient BB84 provides larger operation footprint than conventional BB84
- 4. secret key extraction efficiency enhanced by combining data blocks from several passes.

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Future work:

- 1. More comprehensive constraints to reflect additional restrictions on system operations and deployment
- 2. Incorporate orbital modelling of constellations with cost/performance trade-off studies.
- J. Sidhu, T. Brougham, D. McArthur, R. Pousa, D. Oi, arXiv:2012.07829.

Thank you for your attention!