Robust SDI certification of All Pure Bipartite Maximally Entangled States via Quantum Steering

Harshank Shrotriya, Kishor Bharti, Leong -Chuan Kwek

Introduction

- Quantum device certification will be of paramount importance with the advent of second quantum revolution

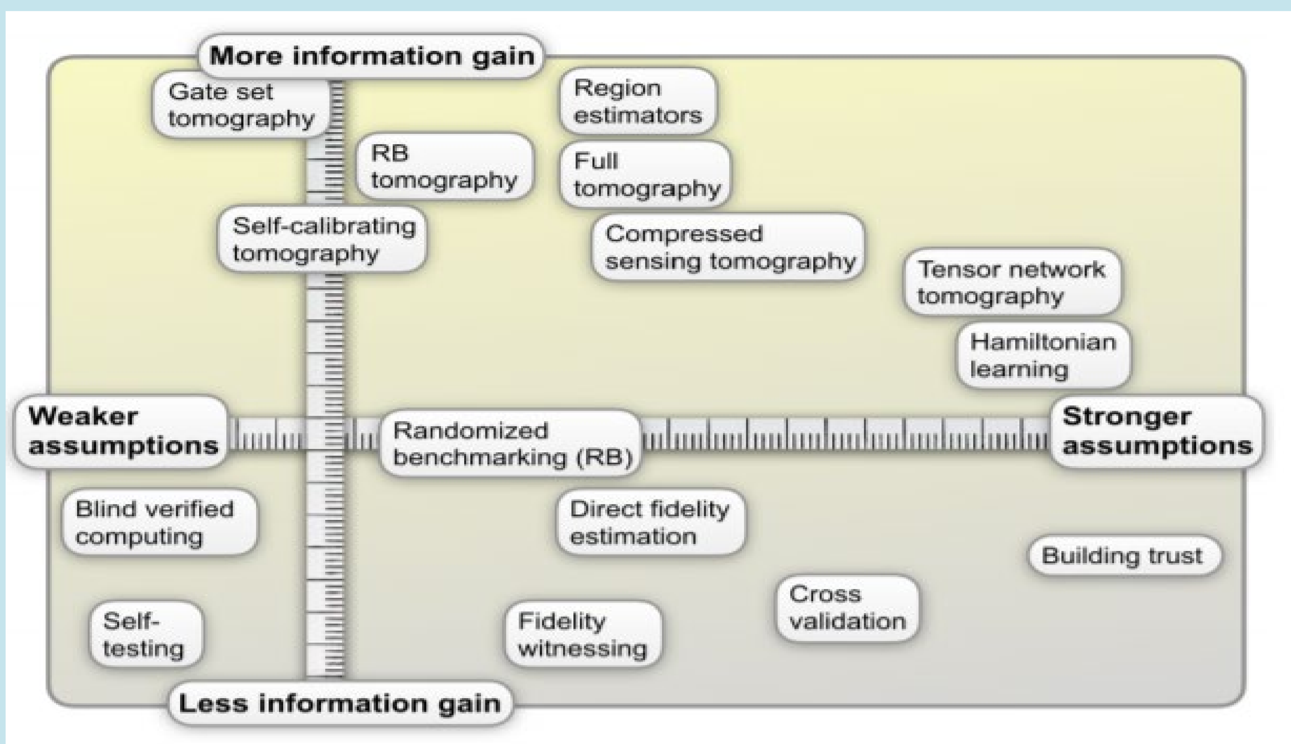

Figure 1: Device Certification Schemes (source: Eisert, J., Hangleiter, D., Walk, N. *et al.* Quantum certification and benchmarking. *Nat Rev Phys* **2,** 382–390 (2020)

- Self Testing is a device certification method requiring weaker assumptions compared to full tomography; however, the state is certified up to local isometry
- Quantum Steering is a phenomenon lying midway between Entanglement and Bell Nonlocality
- Self testing via quantum steering helps in alleviating mathematical difficulties and the semi-device independent scenario arises naturally in cases such as delegated quantum computing
- In this work we show self testing of all pure bipartite states in a semi-device independent scenario using quantum steering

Background

Here we provide some crucial definitions:

- In the semi -device independent scenario, the source generates a state ρ^{AB} and one party (Bob) can fully characterize his side of the state
- However, we treat Alice's measurements as black box measurements, identified by $x = \{0, \dots, m_A - 1\}$ and her outcomes as $a = \{0, \dots, o_A - 1\}$

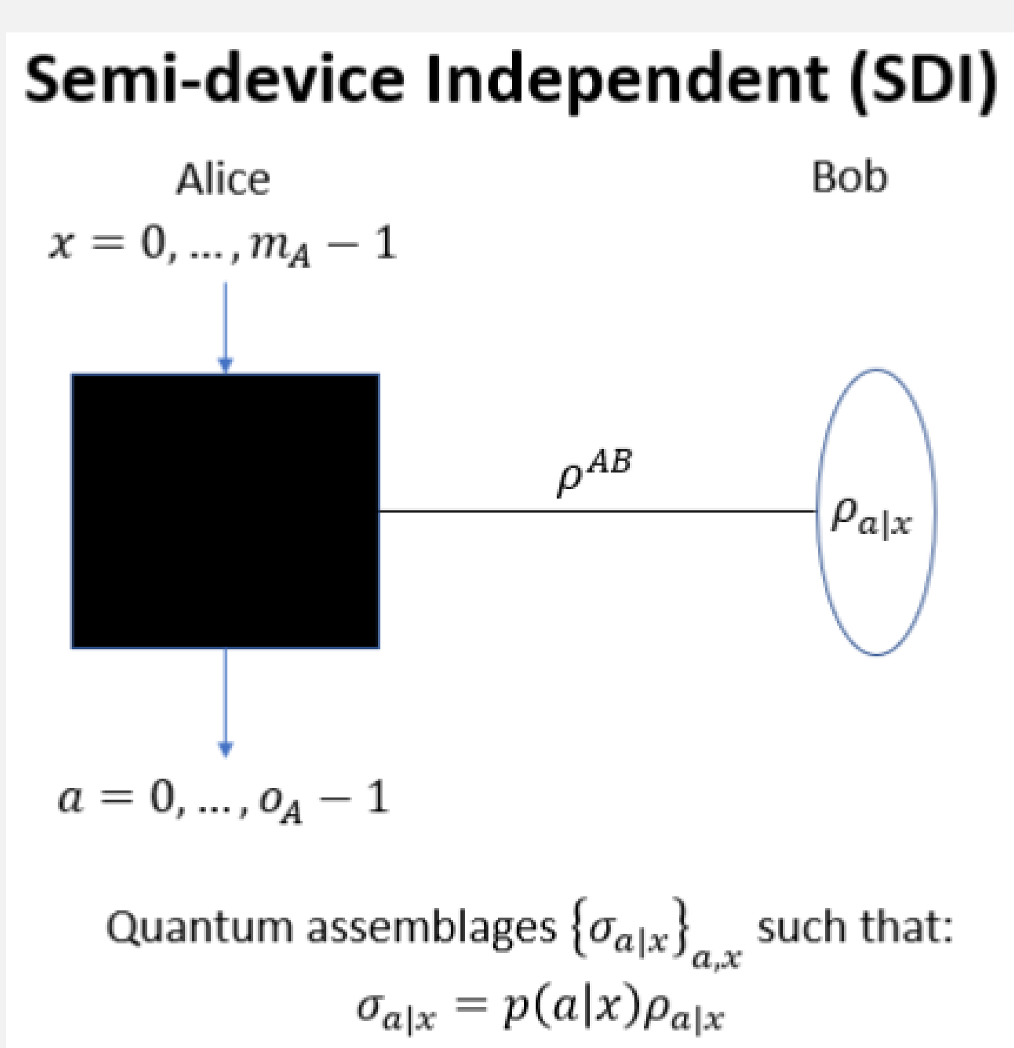

Figure 2: Semi Device independent scenario; quantum assemblages $\{\sigma_{a|x}\}_{a,x}$ are the unnormalized states created on Bob's side after Alice's measurement

- In the SDI scenario, self testing requires the existence of an isometry Φ such that the reference state and measurements $\{|\bar{\psi}\rangle, \bar{M}_{a|x}\}$ on Alice's side can be extracted into the ancilla space from the real state and measurements $\{|\psi\rangle_{AB}, M_{a|x}\}$;

$$\Phi(|\psi\rangle_{AB}) = |junk\rangle_A |\bar{\psi}\rangle_{A'B}$$
$$\Phi(M_{a|x}|\psi\rangle_{AB}) = |junk\rangle_A (\bar{M}_{a|x}|\bar{\psi}\rangle_{A'B})$$

Methods

- Since we are concerned with self testing all pure bipartite entangled states, we can express our target state, using Schmidt decomposition, as:
  $|\psi_{target}\rangle := \sum_{i=0}^{d-1} c_i |ii\rangle$ such that $0 < c_i < 1$ & $\sum_{i=0}^{d-1} c_i^2 = 1$
- To self test this d-dimensional state, we will use subspace projection method as previously done in Ref. $[\mathbf{1},\mathbf{2}]$
- First, we establish a Tilted Steering Inequality the maximal violation of which self tests any *2-qubit* pure entangled state $|\psi\rangle = cos\theta|00\rangle + sin\theta|11\rangle$;
  $$I_{\alpha,\beta} \equiv \alpha\langle A_0\rangle + \beta\langle A_0 Z\rangle + \langle A_1 X\rangle \leq \alpha + \sqrt{1+\beta^2}; \text{where } \beta^2 = 1 + \alpha^2$$
- Then we impose a structure on the ideal set of assemblages $\{\sigma_{a|x}^{ideal}\}_{a,x}$ generated on Bob's side which when observed experimentally self tests the target state $|\psi_{target}\rangle$
- The imposed structure is such that it enables us to certify 2 -dimensional subspace projected blocks of the form $c_{2m}|2m,2m\rangle + c_{2m+1}|2m+1,2m+1\rangle$ of the d-dimensional target state $|\psi_{target}\rangle$

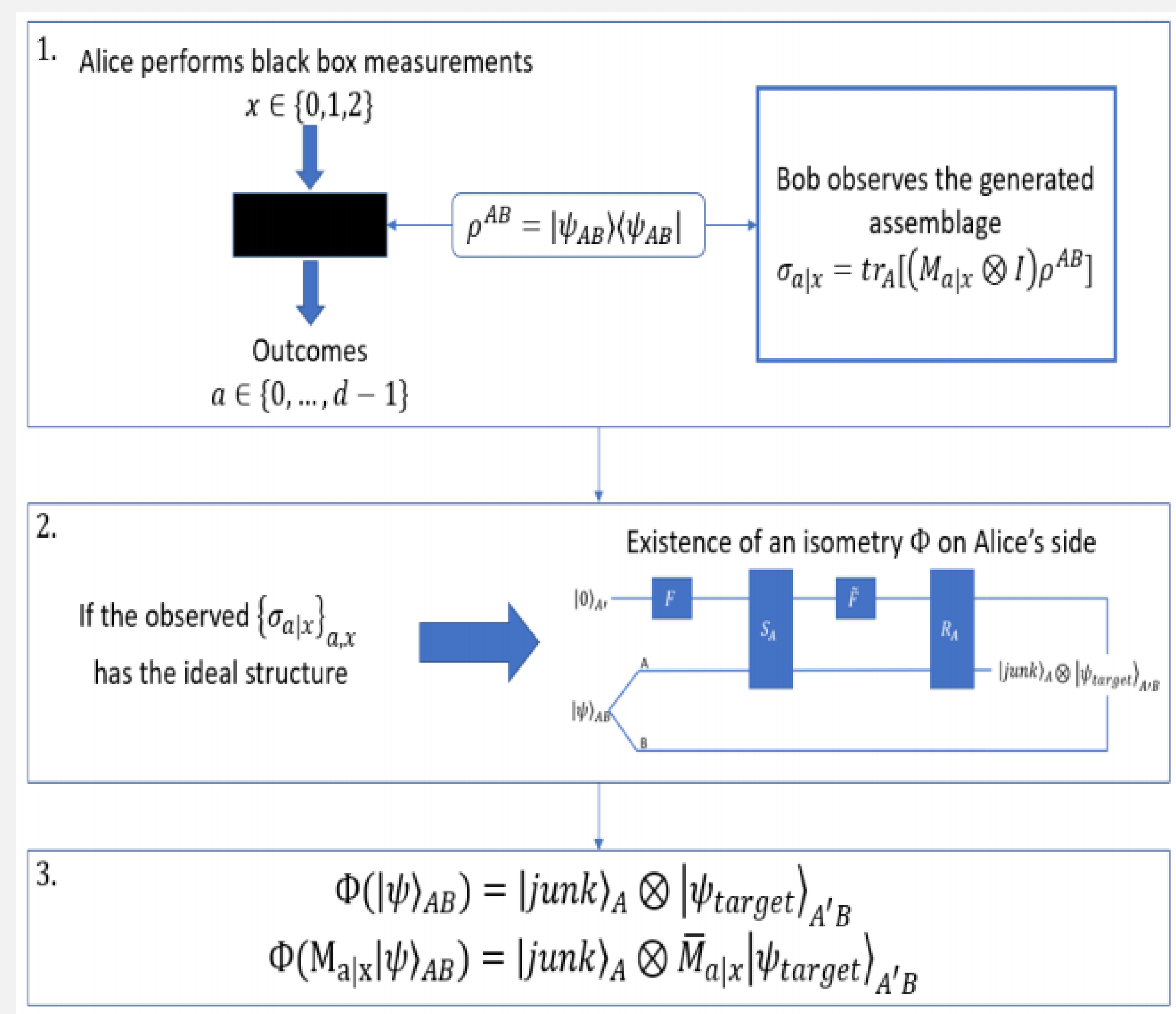

Figure 3: Self Testing scheme

Main Result

- In the SDI scenario, for any bipartite pure entangled state $|\psi_{target}\rangle$, there exists an ideal set of assemblages $\{\sigma_{a|x}\}_{a,x}^{ideal}$, where $x \in \{0,1,2\}$ and $a \in \{0,1,\dots,d-1\}$ which when observed on Bob's side after Alice performs black box measurements on their joint state ρ^{AB} self tests the target state $|\psi_{target}\rangle$ and ideal measurements on Alice's side
- Further, for the case of **maximally** entangled states of any local dimension the SDI certification can be made robust

Acknowledgements

The authors thank the National Research Foundation and the Ministry of Education, Singapore for financial support.

References

1. Yang, T. H., & Navascués, M. (2013). Robust self -testing of unknown quantum systems into any entangled two -qubit states. *Physical Review A* , 87 (5), 050102
2. Coladangelo, A., Goh, K. T., & Scarani, V. (2017). All pure bipartite entangled states can be self -tested. *Nature communications* , 8 (1), 1-5