Fading channel estimation for free-space continuous-variable secure quantum communication

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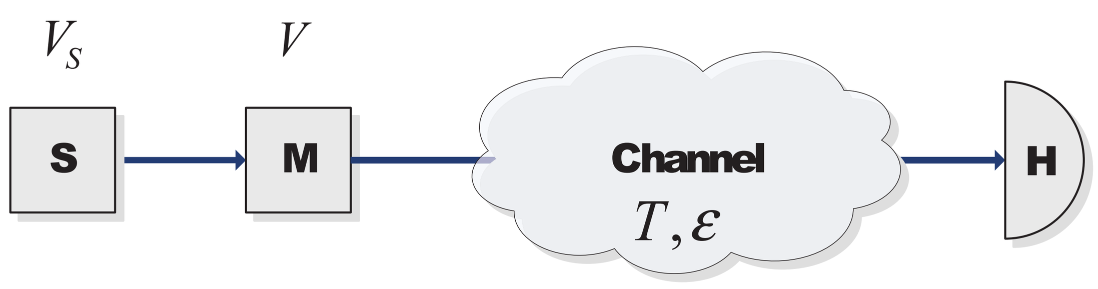
Model and experiment



The model:

- The effect of the channel: $x_B = \sqrt{T} \cdot (x_S + x_M) + \sqrt{1-T} \cdot x_0 + x_\varepsilon$, which can be rewritten as $x_B = \sqrt{T} \cdot x_M + x_N$.
- Key rate: $K = (1-r) \cdot \left[K_\infty(T^{LOW}, V_\varepsilon^{UP}) - \Delta\Big([1-r]N\Big) \right]$, with $K_\infty(T, V_\varepsilon) = \beta I(A:B) - S(B:E)$.

The experiment:

- The experiment was performed in the Erlangen rural area.
- The states were repeatedly sent over with an effective sending rate of $2.48 \cdot 10^6$ states per second $\Rightarrow$ we have a sample size of order 10^7.
- At the remote side the channel transmittance was monitored using a tap-off followed by an intensity measurement.

Effect of fluctuations

- The covariance matrix of outcomes of x_M and x_B is

$$\mathrm{Cov}(x_M, x_B) = \begin{pmatrix} V & \langle\sqrt{T}\rangle V \\ \langle\sqrt{T}\rangle V & \langle T\rangle V' + \varepsilon + 1 \end{pmatrix}$$

- It is equivalent to the fixed channel:

$$\mathrm{Cov}(x_M, x_B) = \begin{pmatrix} V & \sqrt{T_{\mathrm{eff}}} V \\ \sqrt{T_{\mathrm{eff}}} V & T_{\mathrm{eff}} V' + \varepsilon_{\mathrm{eff}} + 1 \end{pmatrix}$$

- The parameters are connected through:

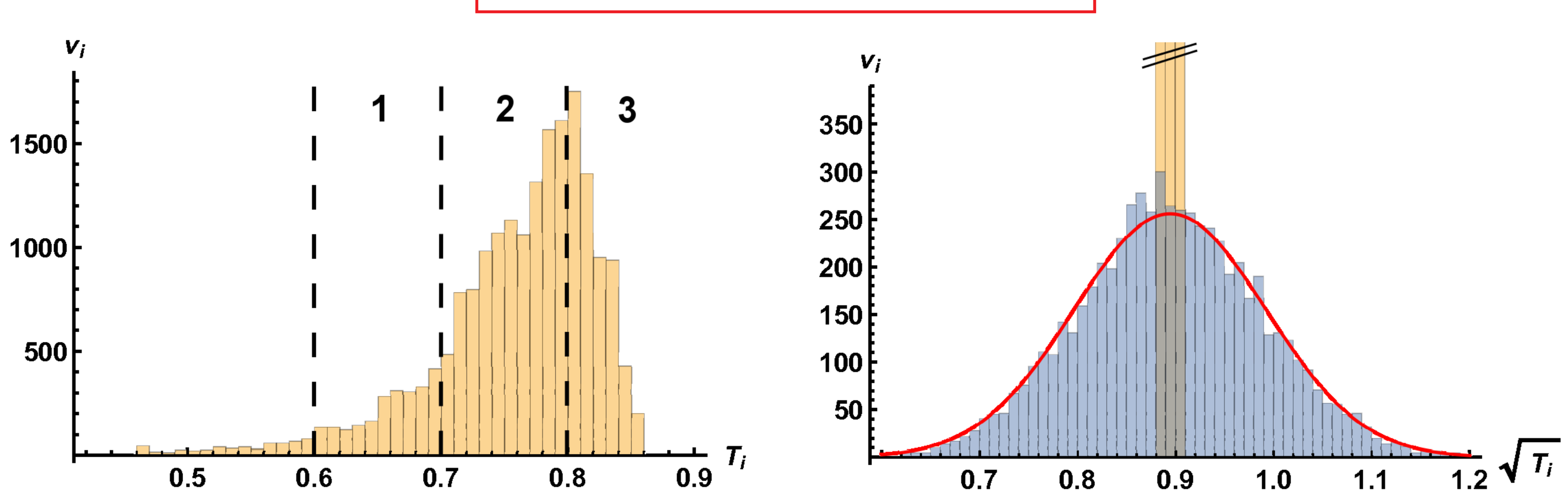
$$T_{\mathrm{eff}} = \langle\sqrt{T}\rangle^2, \quad \varepsilon_{\mathrm{eff}} = \varepsilon + \mathrm{Var}(\sqrt{T}) V'.$$

Package clusterization

The solution for handling a fading channel so far [1] has been to

- discard packages with T_i below a critical value,
- use 99% of states for estimation, 1% of states for communication.

Idea: More clusters! Optimal ratio!

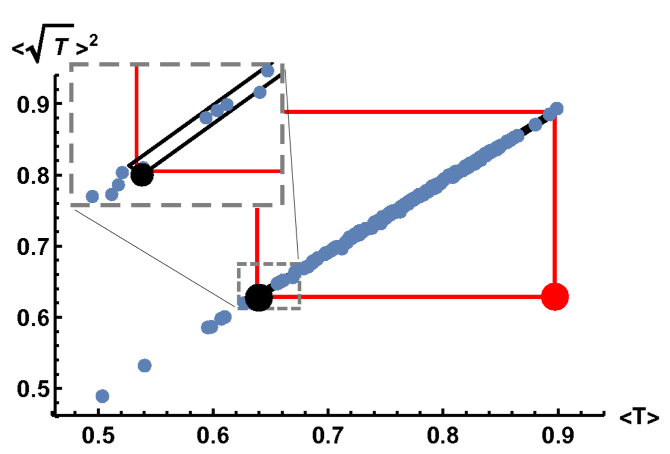


The individual packages can be estimated:

- Estimation of the transmittance $\widehat{\sqrt{T_i}} = \frac{1}{V} \cdot \widehat{C_{MB}}$, with maximum likelihood estimator $\widehat{C_{MB}} = \frac{1}{r \cdot n} \sum_{j=1}^{r \cdot n} M_j B_j$.
- We can obtain the variance of this estimator [2] (see the fit in the right figure):

$$\mathrm{Var}(\widehat{\sqrt{T_i}}) = \frac{1}{r \cdot n} \cdot T_i \left(2 + \frac{V_N}{T_i V}\right)$$

Confidence intervals for channel parameters



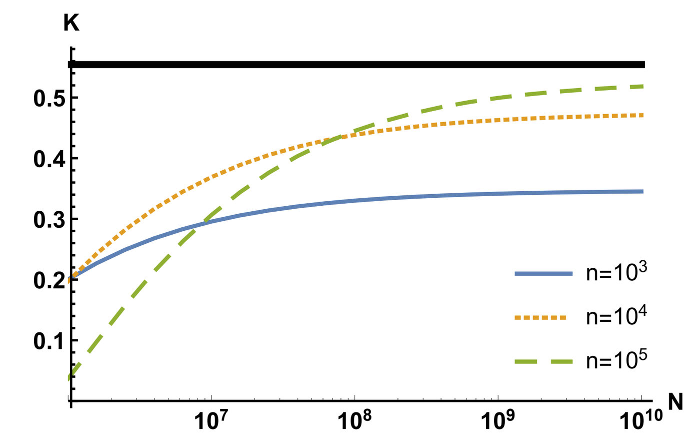
- Worst case (red): lower bound $\langle\sqrt{T}\rangle^{\mathrm{LOW}}$ and upper bounds $\langle T\rangle^{\mathrm{UP}}$ and $\varepsilon^{\mathrm{UP}}$.
- Instead use the transformed variables (black)

$$X_1 := \langle T\rangle - \langle\sqrt{T}\rangle^2 = \mathrm{Var}(\sqrt{T})$$
$$X_2 := \langle T\rangle + \langle\sqrt{T}\rangle^2$$

$\Rightarrow$ 20-30 times better estimation of fluctuation.

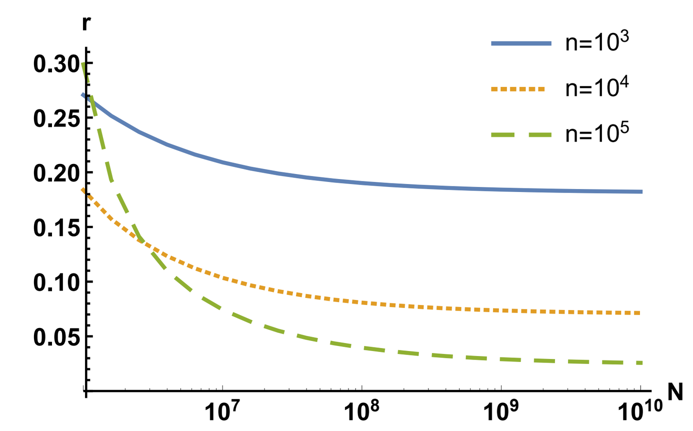
Semi-analytical investigation of the scheme

Dependence on the package size and total number of states



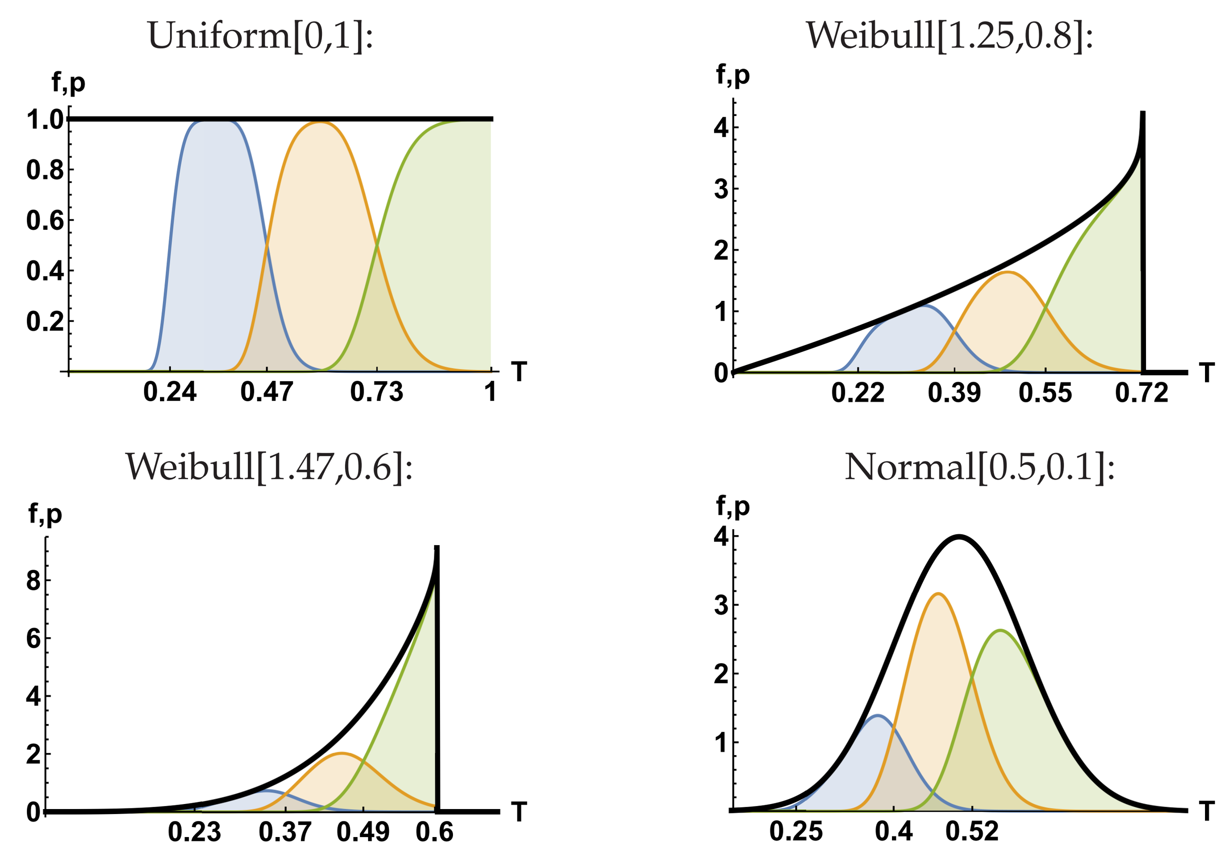
- for finite package sizes the key rate will saturate lower than the key rate of the reference
- the maximally achievable key rate increases with the number of states in each package (n)
- it is also important to have a reasonable number of packages (see the low N values in the figure)

Optimal ratio of states used for estimation

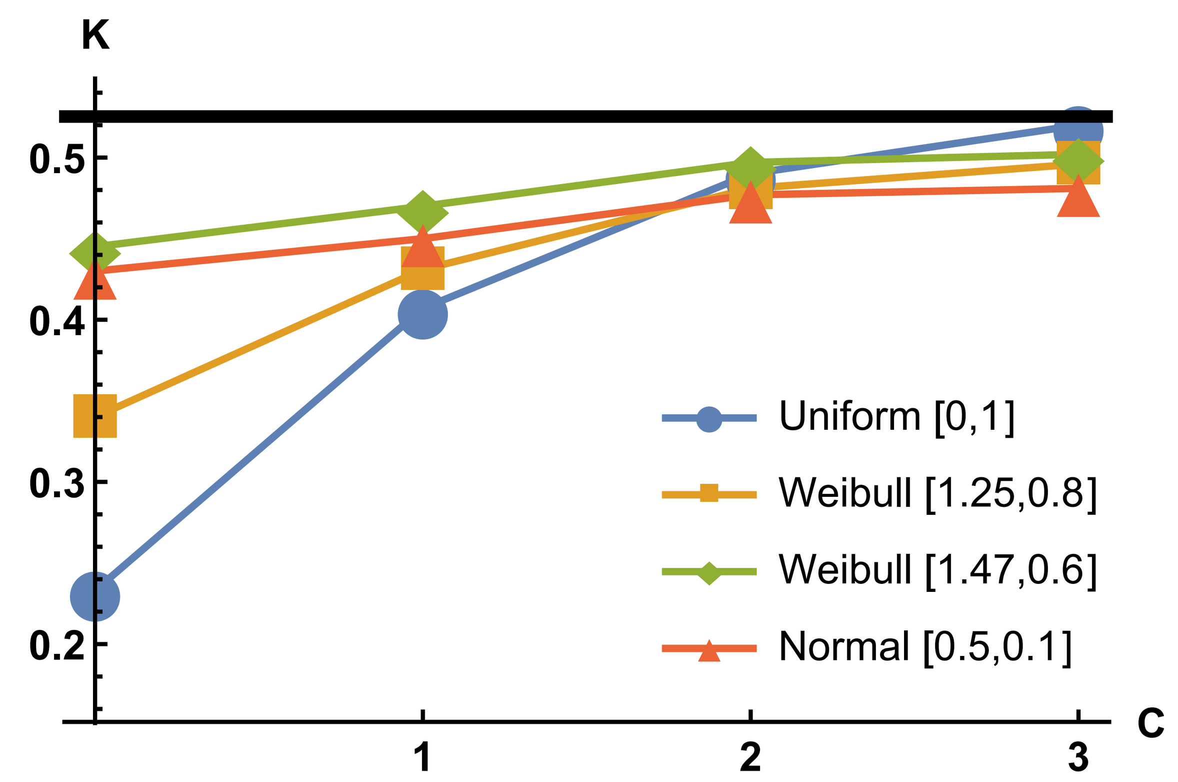


- a lower percentage of states (r) is sufficient for estimation if we have more states
- the values saturate at a non-zero level due to the uncertain estimation for fixed-size packages
- these values are quite low (much lower than the earlier suggested 99.9%)

The ideal clusterization for three clusters ($C = 3$)



The effect of clusterization



Conclusions [3]

- The variance of the estimator can be approximated beforehand $\Rightarrow$ experiment design.
- The optimal ratio of states used for estimation is much lower than in the literature.
- The estimation of the channel is a non-trivial, but important task in realistic QKD settings (especially for fluctuating channels).
- For a lightly fluctuating channel, one can obtain good results even without clusterization.
- For a heavily fluctuating channel, we can get close to the key rate of a fixed-transmission channel by using 2-3 clusters.

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